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422. Proposed by W. D. CAIRNS, Oberlin College.

Find a solution of the equation $x^{x\sqrt{x}} = (x\sqrt{x})^x$. [Adapted from *Godfrey & Siddon's Elementary Algebra.*]

Solved by Edward S. Ingham, F. L. Carmichael, H. T. Biglow, Elizabeth A. Davis. Walter C. Eells, D. H. Richert, H. C. Feemster, A. H. Holmes, Frank Irwin, George Y, Sosnow, Elbert H. Clarke, A. M. Harding, and the Proposer.

Solution by A. M. Harding, University of Arkansas.

If we take the logarithm of both members of the equation, we obtain

$$x\sqrt{x}\log x = x\log(x\sqrt{x}) = x\log(x)^{\frac{3}{2}} = \tfrac{3}{2}x\log x,$$

or

$$x(\sqrt{x} - \tfrac{3}{2})\log x = 0.$$

Hence $x = 0$, $\log x = 0$, and $\sqrt{x} - \frac{3}{2} = 0$. Therefore $x = 0$, $x = 1$, and $x = \frac{9}{4}$. If we substitute $x = 0$, both members of the given equation take the indeterminate form 0^0. In order to show that $x = 0$ is a root, we proceed as follows:

Since $x^{x^{3/2}} = (x^{3/2})^x = x^{\frac{3}{2}x}$, we have $x^{(x^{3/2}-\frac{3}{2}x)} = 1$. Hence we must show that $[x^{(x^{3/2}-\frac{3}{2}x)}] \doteq 1$ as $x \doteq 0$. Let $y = x^{(x^{3/2}-\frac{3}{2}x)}$, then $\log y = (x^{3/2} - \frac{3}{2}x)\log x$, or $\log y = \dfrac{\log x}{(x^{3/2} - \frac{3}{2}x)^{-1}} \doteq \dfrac{\infty}{\infty}$ as $x \doteq 0$. Differentiate numerator and denominator. Then

$$\lim_{x\doteq 0}\frac{\log x}{(x^{3/2} - \frac{3}{2}x)^{-1}} = \lim_{x\doteq 0}\frac{-2(x^{3/2} - \frac{3}{2}x)^2}{3(x^{3/2} - x)} = \lim_{x\doteq 0}\frac{-2x(x^{1/2} - \frac{3}{2})}{3(x^{1/2} - 1)} = 0.$$

Hence $\lim\limits_{x\doteq 0} y = \lim\limits_{x\doteq 0}[x^{(x^{3/2}-\frac{3}{2}x)}] = 1$, and $x = 0$ is a root of the given equation.

423. Proposed by ELBERT H. CLARKE, Purdue University.

Show that the following formula is true for all positive integral values of k. The parenthetical symbols are defined as being the binomial coefficients, and $\binom{k+1}{0} = 1$, by definition.

$$k^k\binom{k+1}{0} - (k-1)^k\binom{k+1}{1} + (k-2)^k\binom{k+1}{2} + \cdots$$
$$+ (-1)^{s-1}(k-s+1)^k\binom{k+1}{s-1} + \cdots + (-1)^{k-1}\binom{k+1}{k-1} = 1.$$

I. Solution by A. M. Harding, University of Arkansas.

There is a misprint in the last term of the left member of the equation as originally given. The exponent should be $k - 1$.

We have

$$e^{kx} = 1 + kx + \frac{k^2x^2}{2!} + \cdots + \frac{k^kx^k}{k!} + \cdots$$

$$e^{(k-1)x} = 1 + (k-1)x + \frac{(k-1)^2x^2}{2!} + \cdots + \frac{(k-1)^kx^k}{k!} + \cdots$$

$$\cdots = \cdots$$

$$e^{(k-s+1)x} = 1 + (k-s+1)x + \frac{(k-s+1)^2x^2}{2!} + \cdots + \frac{(k-s+1)^kx^k}{k!} + \cdots$$

$$\cdots = \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^k}{k!} + \cdots.$$